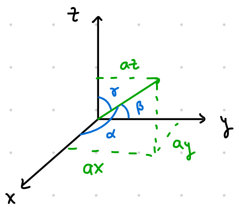
Física I

Teoría

TEMA 1. VECTORES Y MOMENTOS

Operaciones con vectores



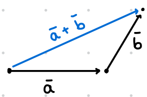
- $\bar{a} = (a_x, a_y, a_z) = a_x \bar{i} + a_y \bar{j} + a_z \bar{k}$
- <u>MÓDULO:</u> $|\bar{a}| = a = \sqrt{a_x^2 + a_y^2 + a_z^2}$
- <u>DIRECCIÓN</u> → vector unitario $\bar{u}_a = \dfrac{\bar{a}}{a}$

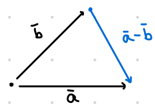
$$\downarrow$$

$$\boxed{\bar{a} = a \cdot \bar{u}_a}$$

- <u>COSENOS DIRECTORES</u>: $\cos\alpha = \dfrac{a_x}{a}$ $\quad \cos\beta = \dfrac{a_y}{a}$ $\quad \cos\gamma = \dfrac{a_z}{a}$

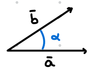
Suma y resta



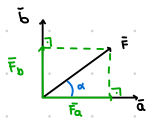


Producto escalar

$\bar{a} \cdot \bar{b} = a_x b_x + a_y b_y + a_z b_z = a \cdot b \cdot \cos\alpha$



- <u>UTILIDADES</u>
  - Proyectar un vector sobre otro



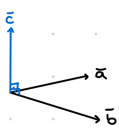
$\bar{F}_a = \left(\bar{F} \cdot \dfrac{\bar{a}}{a}\right) \cdot \dfrac{\bar{a}}{a} = F \cos\alpha \, \bar{u}_a$

$\bar{F}_b = \left(\bar{F} \cdot \dfrac{\bar{b}}{b}\right) \cdot \dfrac{\bar{b}}{b} = F \operatorname{sen}\alpha \, \bar{u}_b$

  - Obtener ángulo entre dos vectores

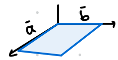
$$\boxed{\cos\alpha = \dfrac{\bar{a} \cdot \bar{b}}{|a||b|}} \qquad \operatorname{sen}^2 + \cos^2 = 1$$

Producto vectorial



$$\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \bar{c} \rightarrow \perp \text{ a } \bar{a} \text{ y a } \bar{b}$$

- Para hallar área de paralelepípedos

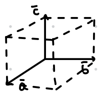


$$\boxed{A = |\bar{a} \times \bar{b}|}$$

- $\bar{a} \times \bar{b} = -\bar{b} \times \bar{a}$

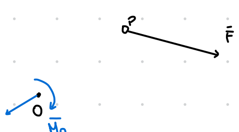
Producto mixto

- Calcular volumen paralelepípedo



$$\boxed{V = |[\bar{a}, \bar{b}, \bar{c}]|} \rightarrow [\bar{a}, \bar{b}, \bar{c}] = \bar{a} \cdot (\bar{b} \times \bar{c}) = \bar{c} \cdot (\bar{a} \times \bar{b}) = \bar{b} \cdot (\bar{c} \times \bar{a}) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

Momento de un vector a un punto

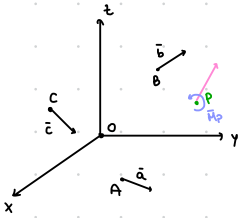


Regla de la mano derecha para saber dirección

$$\boxed{\bar{M}_O = \overline{OP} \times \bar{F}}$$

- O = donde se aplica el momento
- P = punto donde se aplica $\bar{F}$
- $\bar{F}$ = vector que ejerce el momento

Sistemas de vectores



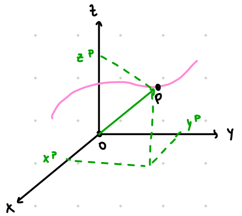
- Sistema de vectores $\rightarrow \{\bar{a}, \bar{b}, \bar{c}\}$
- Reducción de un sistema a un punto P cualquiera:

  $\bar{R}$ = resultante $\quad \bar{R} = \bar{a} + \bar{b} + \bar{c} \dots$

  $\bar{M}_P$ = momento en P $\quad \bar{M}_P = \bar{PA} \times \bar{a} + \bar{PB} \times \bar{b} + \bar{PC} \times \bar{c} \dots$

TEMA 2. CINEMÁTICA DE LA PARTÍCULA

Cinemática en coordenadas cartesianas



- Posición: $\overline{OP} = \bar{r}^P = x^P\bar{i} + y^P\bar{j} + z^P\bar{k}$

  La posición cambia con el tiempo $\longrightarrow$ $\bar{r}(t) = x(t)\bar{i} + y(t)\bar{j} + z(t)\bar{k}$

- Trayectoria: curva que describe la partícula

  Ecuaciones horarias / paramétricas $\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$

  Ecuaciones implícitas $\rightarrow$ 2 ecuaciones
  - se elimina el tiempo de las ecs. paramétricas y se sustituye en las otras

Derivadas

- Notación de derivadas: vector $\bar{a}$ → depende del tiempo t $\bar{a}(t)$; depende de un parámetro θ $\bar{a}(\theta)$

  Derivadas temporales $\longrightarrow$ $\frac{d\bar{a}}{dt} = \dot{\bar{a}}$ $\quad \frac{d^2\bar{a}}{dt^2} = \ddot{\bar{a}}$

  Derivadas paramétricas $\longrightarrow$ $\frac{d\bar{a}}{d\theta} = \bar{a}'$ $\quad \frac{d^2\bar{a}}{d\theta^2} = \bar{a}''$

- Regla de la cadena: teniendo $\bar{a}(\theta)$, buscar $\dot{\bar{a}}$

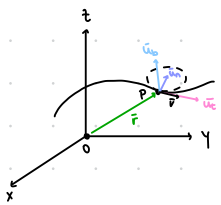
$$\frac{d(\operatorname{sen}\theta)}{dt} = \frac{d(\operatorname{sen}\theta)}{d\theta} \cdot \frac{d\theta}{dt} = \cos\theta \cdot \dot{\theta}$$

Velocidad y aceleración

- Velocidad $\longrightarrow$ $\bar{v} = \frac{d\bar{r}}{dt} = \dot{x}(t)\bar{i} + \dot{y}(t)\bar{j} + \dot{z}(t)\bar{k}$

- Aceleración $\longrightarrow$ $\bar{a} = \frac{d\bar{v}}{dt} = \ddot{x}(t)\bar{i} + \ddot{y}(t)\bar{j} + \ddot{z}(t)\bar{k}$

Coordenadas intrínsecas



Triedro de Frenet

$\bar{u}_t$: unitario tangente $\rightarrow$ tangente a la trayectoria

$\boxed{\bar{u}_t = \frac{\bar{v}}{v}}$ $\longrightarrow$ $\bar{v} = v \cdot \bar{u}_t$

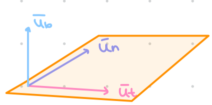
$\bar{u}_b$: unitario binormal $\rightarrow$ $\perp$ a $\bar{u}_t$ y $\bar{u}_n$

$\boxed{\bar{u}_b = \frac{\bar{v} \times \bar{a}}{|\bar{v} \times \bar{a}|}}$

donde $v \rightarrow$ velocidad
$a \rightarrow$ aceleración

$\bar{u}_n$: unitario normal $\rightarrow$ hacia el centro de un círculo tangente

$\boxed{\bar{u}_n = \bar{u}_b \times \bar{u}_t} = -\bar{u}_t \times \bar{u}_b$



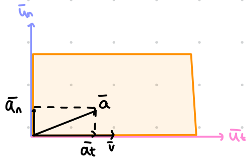
Plano osculador

$\bar{a} \rightarrow$ contenida en plano osculador

$\bar{v} \rightarrow$ dirección de $\bar{u}_t$

- La ecuación de un plano es: $\boxed{A(x - x_0) + B(y - y_0) + C(z - z_0) = 0}$, donde A, B, C son componentes del vector normal ($\perp$) al plano ($\bar{u}_b$) y (x_0, y_0, z_0) es el punto P.

Aceleración tangente y normal

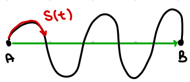


$\boxed{\bar{a} = \bar{a}_t + \bar{a}_n}$

$\boxed{a^2 = a_t^2 + a_n^2}$

- $\bar{v} = v \cdot \bar{u}_t$
- $\boxed{\bar{a}_t = (\bar{a} \cdot \bar{u}_t) \cdot \bar{u}_t} = a_t \cdot \bar{u}_t$ → puede ser ⊕ o ⊖
  $\boxed{a_t = \frac{dv}{dt}}$ → módulo de v · aumenta o disminuye v
- $\boxed{\bar{a}_n = (\bar{a} \cdot \bar{u}_n) \cdot \bar{u}_n} = a_n \cdot \bar{u}_n$ → sólo es ⊕
  $\boxed{\bar{a}_n = \frac{v^2}{\rho}}$ · modifica la dirección de v

Ley horaria



$\boxed{\overline{AB} = \bar{r}_f - \bar{r}_i}$: desplazamiento

S(t) / S(θ) : camino recorrido (ley horaria)

$\boxed{S(t) = \int_0^t |\bar{v}(t)|\, dt}$ $\boxed{S(\theta) = \int_{\theta_0}^{\theta} |v(\theta)|\, d\theta}$

$v = \frac{dS}{dt}$

$a_t = \frac{dv}{dt} = \frac{d^2S}{d^2t}$

Radio de curvatura, curvatura y torsión

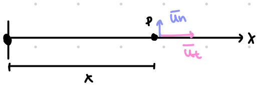
$$\boxed{\rho = \frac{\bar{r}'^3}{|\bar{r}' \times \bar{r}''|} \qquad \rho = \frac{\bar{v}^3}{|\bar{v} \times \bar{a}|}}$$

$$\boxed{K = \frac{|\bar{r}' \times \bar{r}''|}{|\bar{r}'|^3} \qquad K = \frac{|\bar{v} \times \bar{a}|}{|\bar{v}|^3} \qquad K = \frac{1}{\rho}}$$

$$\boxed{\tau = \frac{[\bar{r}', \bar{r}'', \bar{r}''']}{|\bar{r}' \times \bar{r}''|^2}}$$

Movimiento unidireccional

1D

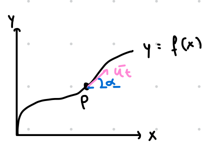


$\bar{r} = \overline{OP} = x\bar{i}$
$\bar{v} = \dot{x}\bar{i}$
$\bar{a} = \ddot{x}\bar{i}$

$\bar{u}_t = \bar{i}$ → $\bar{u}_n = \bar{j}$ en realidad no existe pq es línea recta
$\bar{a} = \bar{a}_t$
$\rho = \infty$ → $\bar{a}_n = 0$
↳ línea recta

Movimiento plano

2D

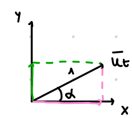


· Trayectoria | $y = f(x)$, $z = 0$ $\tan\alpha = \frac{dy}{dx}$

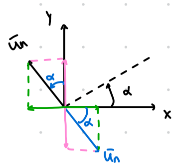
$\frac{1}{\cos^2\alpha} = 1 + tg^2\alpha \longrightarrow \cos\alpha = \frac{1}{\sqrt{1 + tg^2\alpha}}$

$\operatorname{sen}\alpha = tg\alpha \cdot \cos\alpha \longrightarrow \operatorname{sen}\alpha = \frac{tg\alpha}{\sqrt{1 + tg^2\alpha}}$

Proyecciones



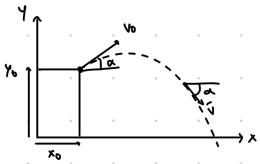
$\bar{u}_t = 1 \cdot \cos\alpha\, \bar{i} + 1 \cdot \operatorname{sen}\alpha\, \bar{j}$



$\bar{u}_n = 1 \cdot -\operatorname{sen}\alpha\, \bar{i} + 1 \cdot \cos\alpha\, \bar{j}$

$\bar{u}_n = 1 \cdot \operatorname{sen}\alpha\, \bar{i} + 1 \cdot -\cos\alpha\, \bar{j}$

Tiro parabólico



$\boxed{\bar{a} = -g\bar{j}}$

$\bar{r_0} = (x_0, y_0)$

$\bar{v_0} = (\dot{x}_0, \dot{y}_0) = (v_0 \cdot \cos\alpha, \ v_0 \cdot \operatorname{sen}\alpha)$

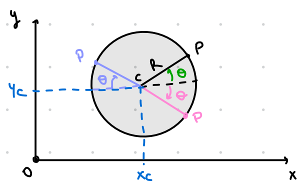
$\bar{v}(t) = \bar{v_0} + \int_0^t \bar{a}\, dt$

$\bar{r}(t) = \bar{r_0} + \int_0^t \bar{v}(t)\, dt$

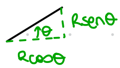
Altura máxima: $\dot{y} = 0 \longrightarrow$ despejar $t^* \longrightarrow y_{max} = y(t^*)$

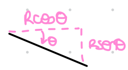
Alcance: $D = x_f - x_0 \longrightarrow x_f \longrightarrow y = 0 \longrightarrow$ despejar $t^* \longrightarrow x_f = x(t)$

Movimiento circular



$\bar{OP} = \bar{r} = \bar{OC} + \bar{CP} = (x_c, y_c) + (R\cos\theta, \ R\operatorname{sen}\theta) = (x_c + R\cos\theta, \ y_c + R\operatorname{sen}\theta)$





$+ (R\cos\theta, \ -R\operatorname{sen}\theta) = (x_c + R\cos\theta, \ y_c - R\operatorname{sen}\theta)$

$+ (-R\cos\theta, \ R\operatorname{sen}\theta) = (x_c - R\cos\theta, \ y_c + R\operatorname{sen}\theta)$

$\boxed{r = R\theta}$

- $\bar{v} = \frac{d\bar{r}}{dt} = \frac{d}{dt}(x_c + R\cos\theta, \ y_c + R\operatorname{sen}\theta) = (-R\dot{\theta}\operatorname{sen}\theta, \ R\dot{\theta}\cos\theta) = R\dot{\theta}(-\operatorname{sen}\theta, \cos\theta)$
  $\hookrightarrow \boxed{v = R\dot{\theta}}$

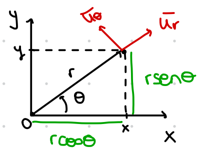
- $\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}(R\dot{\theta}(-\operatorname{sen}\theta, \cos\theta)) = \underbrace{R\ddot{\theta}(-\operatorname{sen}\theta, \cos\theta)}_{\bar{a}_t} + \underbrace{R\dot{\theta}^2(-\cos\theta, -\operatorname{sen}\theta)}_{\bar{a}_n}$

$\boxed{a_t = R\ddot{\theta}}$

$a_n = \frac{v^2}{\rho} \longrightarrow \bar{a} = \bar{a}_t + \bar{a}_n$

$\boxed{a_n = \frac{(R\dot{\theta})^2}{R} = R\dot{\theta}^2}$

Cinemática en coordenadas polares



- CARTESIANAS: $\bar{r} = r\cos\theta\,\bar{i} + r\operatorname{sen}\theta\,\bar{j}$

$\begin{cases} x = r\cos\theta \\ y = r\operatorname{sen}\theta \end{cases} \longrightarrow \begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \operatorname{arctg}(y/x) \end{cases}$

- POLARES: $\bar{r} = \bigcirc\,\bar{u}_r + \bigcirc\,\bar{u}_\theta$

Para pasar de polares a cartesianas se proyecta:

$\bar{u}_r = \cos\theta\,\bar{i} + \operatorname{sen}\theta\,\bar{j}$

$\bar{u}_\theta =$
- antihorario $\longrightarrow -\operatorname{sen}\theta\,\bar{i} + \cos\theta\,\bar{j}$
- horario $\longrightarrow \operatorname{sen}\theta\,\bar{i} - \cos\theta\,\bar{j}$

- Posición en polares: $\boxed{\bar{r} = r\bar{u}_r}$

- Velocidad en polares:

$\bar{v} = \frac{d\bar{r}}{dt} = \frac{d}{dt}(r\bar{u}_r) = \dot{r}\bar{u}_r + r\underbrace{\frac{d\bar{u}_r}{dt}}_{\text{al derivar un unitario, no cambia el módulo; sólo la dirección}} = \boxed{\overbrace{\dot{r}\bar{u}_r}^{v_r \text{ (v radial)}} + \overbrace{r\dot{\theta}\bar{u}_\theta}^{v_\theta \text{ (v azimutal)}} = \bar{v}}$

- aceleración en polares:

$\boxed{\bar{a} = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{a_r}\bar{u}_r + \underbrace{(r\ddot{\theta} + 2\dot{r}\dot{\theta})}_{a_\theta}\bar{u}_\theta}$